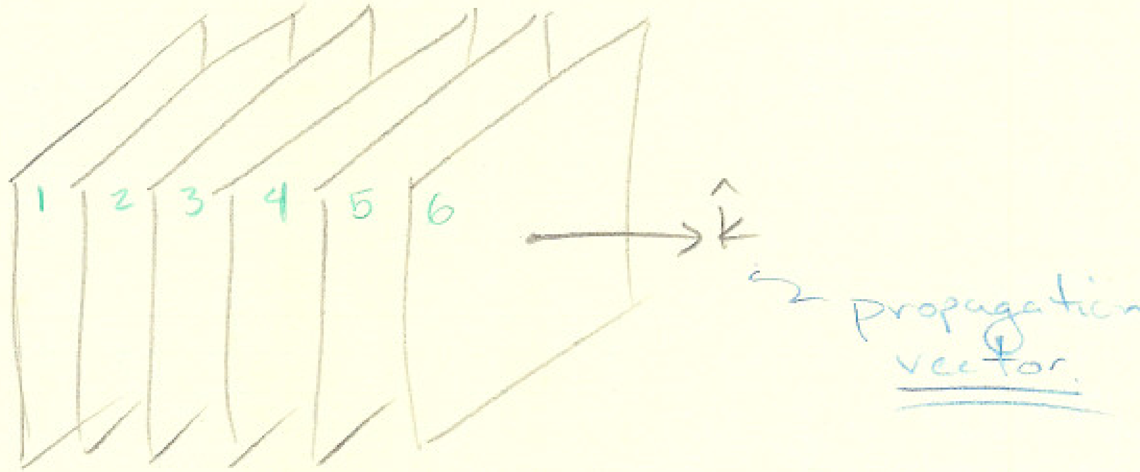
Q: What is a plane wave? ($\vec{k} \cdot r$ is constant)

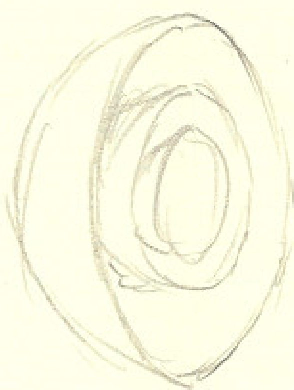
A:



Q What is a sphereical wave?

($\vec{k} \cdot \hat{r}$ is constant on sphere)

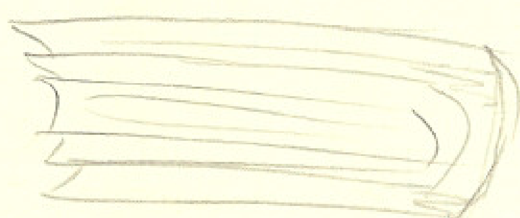
A

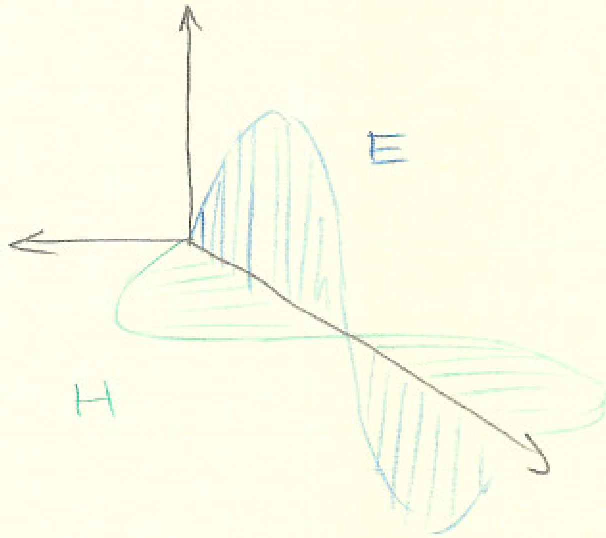


$\hat{k}$ is $\hat{r}$

Q What is a cylindical wave?

A





Amplitude relations

$|\vec{E}| = V/m$

$|\vec{H}| = A/m$

$\therefore \quad \frac{|\vec{E}|}{|\vec{H}|} = \frac{V}{I} = R$

$$\vec{\nabla} \times \vec{E}_s = -j\omega\mu \vec{H}_s$$

Specialise again WLOG (without loss of generality) to $\vec{E}_s$ along $\hat{x}$ and $\vec{k}$ along $\hat{z}$ ... therefore $\vec{H}_s$ is along $\hat{y}$

$$\vec{E}_s = \hat{x} E_{os} e^{-\alpha z} e^{-j\beta z}$$

$$\vec{H}_s = -\hat{y}(\alpha + j\beta) E_{os} e^{-\alpha z} e^{-j\beta z}$$

$$= -j\mu\omega \vec{H}_s$$

$$\vec{H}_s = \hat{y}(-j\omega\mu) H_{sy}$$

$$-(\alpha + j\beta) E_{ox} = -j\mu\omega H_{sy}$$

$$H_{sy} = \left(\frac{\beta - j\alpha}{\omega\mu}\right) E_{ox} = \frac{1}{\eta_c} E_{ox}$$

$$\left.\frac{1}{\eta_c} = \frac{\beta - j\alpha}{\omega\mu}\right\} \text{ 1/impedance}$$

Note: the impedance is complex, $1/\eta_c$ is real when $\alpha = 0$.

$$\vec{H}_s = \hat{y}\frac{1}{\eta_c} E_{ox} e^{-\alpha z} e^{-j\beta z}$$

The "y" amplitude of $\vec{H}_s$ is connected to the "x" amplitude of $\vec{E}_s$

$$\eta_c = |\eta_c| e^{j\phi_n}$$

$$E_{ox} = |E_{ox}| e^{j\phi_x}$$

$$\vec{E}(z,t) = \mathrm{Re}\left[\hat{x}|E_{ox}| e^{j\phi_x} e^{-\alpha z} e^{-j\beta z} e^{j\omega t}\right]$$

$$= \hat{x}|E_{ox}| e^{-\alpha z}\cos(\omega t - \beta z + \phi_x)$$

$$\vec{H}(z,t) = \hat{y}\frac{|E_{ox}|}{|\eta_c|} e^{-\alpha z}\cos(\omega t - \beta z + \phi_x - \phi_n)$$

when $\alpha = 0$ then $\sigma = 0$.

EX:

$$\mu_o = 4\pi \times 10^{-7}$$

$$\varepsilon_o = 8.85 \times 10^{-12}$$

$$\sigma = 0$$

$$\alpha = 0$$

$$\beta = \sqrt{\omega^2 \mu_o \varepsilon_o}$$

$$\eta = \eta_o = \frac{\omega \mu_o}{\beta} = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

$$\simeq 376.6\,\Omega = 377\,\Omega$$

$$= 120\pi$$

Vacuum is lossless: $\boxed{\begin{array}{ll} \sigma = 0 \ \therefore \ \alpha = 0 \\ \alpha = 0 \ \therefore \ \sigma = 0 \end{array}}$

$$\beta = \sqrt{\omega^2 \mu_o \varepsilon_o} \qquad \alpha = 0 \qquad \boxed{\eta = \sqrt{\frac{\mu}{\varepsilon}}}$$

real

Low loss dielectrics

Recall:

$$\alpha = \sqrt{\frac{\mu \varepsilon \omega^2}{2}} \left[\left(1 + \frac{\sigma^2}{\omega^2 \varepsilon^2} \right)^{1/2} - 1 \right]^{1/2}$$

$$\beta = \sqrt{\frac{\mu \varepsilon \omega^2}{2}} \left[\left(1 + \frac{\sigma^2}{\omega^2 \varepsilon^2} \right)^{1/2} + 1 \right]^{1/2}$$

low loss results when

$$\frac{\sigma}{\omega \varepsilon} \lll 1$$

Numerically

$$\frac{\sigma}{\omega\varepsilon} < 1/100$$

Note that this "low loss" is dependent on freq.

Recall: The binomial theorem

$$\boxed{\begin{array}{l}\text{if "}x\text{" is "small", then} \\ (1+x)^n \sim 1 + nx + \dots\end{array}}$$

$$\therefore \quad x = \frac{\sigma^2}{\omega^2\varepsilon^2} = \left(\frac{\sigma}{\omega\varepsilon}\right)^2$$

$$\therefore \quad (1+x)^{1/2} = 1 + \frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2$$

$$\alpha \rightarrow \sqrt{\frac{\omega^2\mu\varepsilon}{2}}\left[1 + \frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2 - 1\right]^{1/2}$$

$$= \sqrt{\frac{\omega^2\mu\varepsilon}{2} \cdot \frac{1}{2}\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$$

$$\beta \rightarrow \sim \sqrt{\frac{\omega^2\mu\varepsilon}{2}} \cdot \sqrt{2} = \sqrt{\omega^2\mu\varepsilon}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Good conductors

$$\frac{\sigma}{\omega\varepsilon} >>> 1$$

$$\alpha \cong \sqrt{\frac{\omega^2\mu\varepsilon}{2}}\left[\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{1/2} \mp 1\right]^{1/2}$$

$$\cong \sqrt{\frac{\omega^2\mu\varepsilon}{2}}\left[\frac{\sigma}{\varepsilon\omega} - 1\right]^{1/2}$$

$$\cong \sqrt{\frac{\omega^2\mu\varepsilon}{2}\,\frac{\sigma}{\varepsilon\omega}}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}$$

$\beta \rightarrow$ do the same

$= \alpha$

$$\eta_c = \frac{1}{\sqrt{2}}(1+j)$$